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A Second Generation of Backstepping Designs and Robust Nonlinear Control of Aeroengines

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FINAL REPORT

SUBMITTED TO:

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Air Force Office of Scientific Research

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Abstract

This technical report reviews the research conducted during a 14.5 month period while the PI was on the faculty at University of Maryland. This research has been continued at the PI's new institution, University of California, San Diego, under the number F08671-9800319.

The objective of this research has been twofold: (1) to develop nonlinear control design techniques that can systematically accommodate nonlinearities and have optimality properties, and (2) develop control methods for robust stabilization and optimization of performance of aeroengine compressor systems.

1. NONLINEAR CONTROL THEORY.

- (a) *Optimal Control of Nonlinear Systems with Uncertainties.* Our most significant theoretical results have been on the subject of developing adaptive and robust controllers for nonlinear systems. This is a subject well known for its difficulty—most existing results are only existence results which do not provide a method for *designing* a controller. We have developed the first analytical procedure for designing (closed-form) optimal adaptive controllers. For this result, a student funded on this project, Zhonghua Li, received the Best Student Paper Award at the 1997 American Control Conference. We have also addressed problems with time-varying disturbances and found a closed-form solution to a problem similar to, but more general than, the nonlinear \mathcal{H}_∞ (differential game) problem, thus avoiding the need to solve Hamilton-Jacobi-Isaacs PDE's.
- (b) *Robustification of Adaptive Nonlinear Control.* (A) For nonlinear systems with control singularities, e.g., systems where feedback linearization designs result in dramatically reduced regions of attraction, we have developed a backstepping design which maximizes the region of attraction; this is achieved by designing control Lyapunov functions whose singularity set coincides with the singularity set of the control. (B) We have continued our study of asymptotic properties of adaptive nonlinear systems (for which the PI received the O. Hugo Schuck Best Paper Award at the 1995 American Control Conference; following our initial result that, in the case of a vanishing regressor, parameter estimates may converge to destabilizing values but only from a set of initial conditions of zero measure, we have obtained a surprising result that in the case of a partially vanishing regressor this set may have *nonzero measure*; this result disproves a decades-long intuition that robustness of adaptive feedback loops increases monotonically with the amount of excitation. (C) The results on the subject of *robustness* of adaptive nonlinear backstepping designs includes redesigns of tuning functions and modular schemes for robustness to bounded uncertainties and a redesign of a scheme for linear systems for robustness to a very general class of dynamic uncertainties including those that reduce the relative degree and introduce unstable zeros.

2. CONTROL OF AEROENGINE COMPRESSORS.

- (a) *Stabilization of Rotating Stall and Surge.* We have developed a family of robust nonlinear control laws for the cubic Moore-Greitzer model. These control laws

employ only 1D sensing and require only the knowledge of the maximal slope of the axisymmetric compressor characteristic. This is achieved using the backstepping methodology which allows us to avoid cancellation of useful nonlinearities, thereby reducing the required control effort. In contrast to previous results which change the bifurcation from subcritical to supercritical but achieve only *local* stability, our design also achieves *global* stability. In addition to studying the MG *cubic* model, we have developed a control methodology (a) for compressors with a more realistic "deep-hysteresis" (right-skew) characteristic, (b) with actuators of limited bandwidth, and (c) with a high value of Greitzer's B parameter.

- (b) *Maximization of Pressure Rise.* We have addressed the important practical problem of maximizing the pressure rise, while simultaneously stabilizing rotating stall and surge. A compressor in which stall and surge are stabilized has an operating characteristic that has a maximum, but the characteristic, and the location of its maximum, are unknown. Our "extremum seeking" scheme employs a periodic perturbation signal and filters which steer the operating point to maximum pressure rise from any initial condition. Besides providing design guidelines for such a scheme, we have proved analytically that the scheme achieves desired convergence and stability using averaging and singular perturbation theory.

Contents

1	Introduction	5
2	Nonlinear Control Theoretic Results	5
2.1	Synthesis of Nonlinear Optimal Controllers for Systems with Deterministic Uncertainties	5
2.2	Geometric Structure, Invariant Manifolds, and Asymptotic Properties of Adaptive Nonlinear Controllers	6
2.3	Maximizing Regions of Attraction for Nonlinear Systems with Control Singularities	6
2.4	Robust Redesigns of Adaptive Nonlinear Controllers	7
2.5	Input Unmodeled Dynamics	7
3	Compressor Control	7
3.1	Rotating Stall and Surge in Jet Engine Compressors	7
3.2	"Extremum Seeking" for General Nonlinear Systems and Maximization of Compressor Pressure Rise	8
4	Other Applications	8
4.1	Optimal Control of Spacecraft	8
4.2	Aircraft Wing Rock	8
5	PUBLICATIONS	9
6	Additional Data	12
6.1	Personnel Supported	12
6.2	Transitions	12
6.3	Patent Disclosures	12
6.4	Awards	12

1 Introduction

The state of the art in nonlinear control at the beginning of this research was such that control schemes were available for stabilization of certain classes of nonlinear systems but these schemes were often too complex to synthesize, required a high control effort, and their robustness was not guaranteed. This research was concerned with establishing connections between systematic methods of nonlinear stabilization and optimal control. As our main result, we provide redesigns of adaptive and robust nonlinear controllers which are inverse optimal, and thus use less control effort and possess a certain margin of robustness to some uncertainties.

The objective of the theoretical research is an application to nonlinear instabilities arising in aeroengine compressor systems. We have advanced the state of the art in this area in two directions. First, we have developed the first nonlinear control laws with guaranteed regions of attraction and robustness to some modeling errors. Second, we have pioneered the methods for on-line seeking of the maximum of the compressor pressure rise characteristic.

The results of our work have been published (or submitted for publication) in 15 journal papers, 10 conference papers, and two book chapters. They are discussed in three major sections of this technical report, Section 2 which discusses our theoretical accomplishments, Section 3 which discusses our applications to aeroengine compressors, and Section 4 which discusses our other applications.

This research is continuing and has undergone additional research advances, as well as additional progress in transition, since the termination of the present grant in August 1997. These advances will be reported on in the final report for the new grant.

2 Nonlinear Control Theoretic Results

2.1 Synthesis of Nonlinear Optimal Controllers for Systems with Deterministic Uncertainties

Nonlinear optimal control problems reduce to Hamilton-Jacobi pde's which are difficult to solve. In [10] and [9], we have systematically approached nonlinear systems affine in disturbances or parametric uncertainties and provided analytical solutions to HJ pde's corresponding to meaningful cost functionals that incorporate positive definite, radially unbounded, penalties on the state and control.

1. **Disturbances—Differential Game Approach** In [10] we proved that input-to-state stabilizability (as defined by Sontag) is both *necessary and sufficient* for the solvability of an Isaacs equation associated with a meaningful differential game problem. Our cost functional is more general than in the “nonlinear \mathcal{H}_∞ ” problem and the penalties on the disturbance and the control in general become non-quadratic (rather than completion of squares, our main technical tool are Legendre-Fenchel transforms), which allows an analytical solution to the pde. We present analytical constructions of optimal controllers using input-to-state stabilization techniques and backstepping.

2. **Parametric Uncertainties—Adaptive Approach** In [9] we proved that an inverse optimal adaptive tracking problem is solvable if and only if an “adaptive control Lyapunov function” exists for the system. The adaptive controller that we present is the first adaptive controller that is truly optimal. Previous certainty-equivalence adaptive controllers for linear systems, based on solving a Riccati equation for each value of the parameter estimate, do not lead to optimality of the controller with respect to the overall plant-estimator system. Our controllers achieve optimality by compensating for the effect of parameter adaptation transients. For controllers in [9] we also derive transient performance bounds that include an estimate of control effort, which is the first such result in the adaptive control literature. For [9] the student Zhonghua Li (funded by this grant) received the Best Student Paper Award at the 1997 American Control Conference.

2.2 Geometric Structure, Invariant Manifolds, and Asymptotic Properties of Adaptive Nonlinear Controllers

Since the beginnings of adaptive control in the 1950's, a fundamental question that has remained open is whether the parameter estimates always converge to stabilizing values. In fact, except for the case of persistent excitation or least-squares estimation, it was not even known if the parameter estimates converge at all. In [2], we provided an affirmative answer to the question of convergence to constant estimates, and then, using center manifold and invariant manifold theorems, showed that the parameter estimates *do* converge to stabilizing values, except, somewhat surprisingly, starting from a set of initial conditions of Lebesgue measure zero. This enabled us to further prove that, after sufficiently long time, the adaptation can be disconnected without destroying the closed-loop system stability, which is important in practice where, after long adaptation, the identifier no longer ‘follows’ the system but the noise. These results [2] received the 1996 O. Hugo Schuck Paper Award of the American Automatic Control Council.

In a subsequent study in conditions of “partial excitation” [8] we revealed another surprising result by constructing a simple example where the controller converges to destabilizing estimates from initial conditions of *positive measure*. This example disproves the general intuition in adaptive control (drawn from the well-founded benefits of persistent excitation) that a *partially exciting* regressor is more favorable than a *vanishing* regressor.

2.3 Maximizing Regions of Attraction for Nonlinear Systems with Control Singularities

For nonlinear systems with control singularities the region of attraction is often only a subset of the region of feasibility (nonsingularity) of the control law. In [4] we presented conditions under which the backstepping design makes the regions of feasibility and stability coincide. The key element is a control Lyapunov function which is singular on the control singularity set and has level sets that always remain in the feasibility region, thus making the feasibility region positively invariant.

2.4 Robust Redesigns of Adaptive Nonlinear Controllers

Even for linear plants, benign-looking disturbances and unmodeled dynamics can cause unbounded solutions. We are currently trying to address these problems for adaptive *nonlinear* systems. In [5,7] we showed that the standard robustification tool—dynamic normalization—is not necessary in tuning functions schemes. Instead, we employ only leakage or parameter projection to prevent the drift of the estimates. Because of the nonlinear character of the controller, robustness results for the tuning functions schemes are not global but regional, with a region of attraction inversely proportional to the "size" of the unmodeled dynamics.

In addition to unmodeled dynamics, we have also addressed the problem of bounded uncertainties in [6]. For each of the previously developed adaptive nonlinear designs, we presented a redesign which guarantees *global* boundedness of solutions and arbitrarily small \mathcal{L}_2 and \mathcal{L}_∞ gains with respect to the disturbances.

2.5 Input Unmodeled Dynamics

Most of available results in robust nonlinear control consider problems of uncertain static nonlinearities and external disturbances. The problem of unmodeled dynamics has recently been addressed by Jiang, Teel, and Praly using their nonlinear small-gain tools, but not for unmodeled dynamics at the input of the system. In [1] we addressed a broad class of nonlinear systems dynamically perturbed at the input, as well as at various other locations throughout the system, and designed robustifying feedback laws that employ a novel tool of *dynamic nonlinear damping*. This result has been followed by several extensions and generalizations by other authors.

3 Compressor Control

3.1 Rotating Stall and Surge in Jet Engine Compressors

Compressor stall and surge are complex nonlinear instabilities that reduce the performance and can cause failure of aircraft engines. In [11] we designed a feedback controller that globally stabilizes a broad range of possible equilibria in a nonlinear compressor model of Moore and Greitzer. This is achieved with a novel type of backstepping design which avoids cancellation of the system's useful nonlinearities—those that would be cancelled in a feedback linearizing design. While stability analysis is involved, the resulting control law is simple and optimal with respect to a meaningful non-quadratic cost functional. As in a previous bifurcation-theoretic design by Liaw and Abed, we change the character of the bifurcation from subcritical to supercritical. However, since we do not use bifurcation normal forms but Lyapunov tools, our controller achieves not only local but also global stability. The controller requires minimal modeling information (bounds on the slope of the stall characteristic and the B -parameter) and simpler sensing (rotating stall is stabilized without measuring its amplitude).

In a two-part article [14,15] we have also proposed controllers (a) for compressors with a more realistic "deep-hysteresis" characteristic, (b) with actuators of limited bandwidth, and (c) with a high value of Greitzer's B parameter. We also generated a parametrization of

deep-hysteresis models which uses a single scalar parameter to span a family of compressors with an increasing depth of hysteresis.

3.2 “Extremum Seeking” for General Nonlinear Systems and Maximization of Compressor Pressure Rise

While the mainstream methods of adaptive control (both linear and nonlinear) deal only with regulation to *known* set points or reference trajectories, in many applications the set point should be selected to achieve a maximum of an *uncertain* reference-to-output equilibrium map. The techniques of the so-called “extremum control” or “self-optimizing control” developed for this problem in the 1950–1960’s have long gone out of fashion in the theoretical control literature because of the difficulties that arise in a rigorous analytical treatment. In [12] we provide the first proof of stability of an extremum seeking feedback scheme by employing the tools of averaging and singular perturbation analysis. Our scheme is much more general than the existing extremum control results which represent the plant as a static nonlinear map possibly cascaded with a linear dynamic block—we allow the plant to be a general nonlinear dynamic system (possibly non-affine in control and open-loop unstable) whose reference-to-output equilibrium map has a maximum, and whose equilibria are locally exponentially stabilizable. We apply the extremum seeking scheme to a nonlinear model of an aeroengine compressor and maximize the steady-state pressure rise, while also stabilizing rotating stall and surge.

4 Other Applications

4.1 Optimal Control of Spacecraft

Optimal regulation of attitude and velocity in spacecraft models is a problem that has been open since the 1960’s. We have recently solved it in [13] via inverse optimality combined with backstepping. Our cost functional puts a nonquadratic penalty on the angular velocity, orientation, and control torque to account for the nonlinearity of the system.

4.2 Aircraft Wing Rock

Wing rock is a nonlinear oscillation in the roll angle caused by asymmetric leading-edge vortices in fighter aircraft flying at high angle-of-attack. We developed an adaptive nonlinear controller [3] which achieves attenuation of oscillations on a wing-rock model with uncertain aerodynamic parameters and “rolling-moment derivative.”

5 PUBLICATIONS

Journal Papers

(appeared)

- [1] M. Krstić, J. Sun and P. V. Kokotović, "Robust control of nonlinear systems with input unmodeled dynamics," *IEEE Transactions on Automatic Control*, vol. 41, pp. 913-920, 1996.
- [2] M. Krstić, "Invariant manifolds and asymptotic properties of adaptive nonlinear stabilizers," *IEEE Transactions on Automatic Control*, vol. 41, pp. 817-829, 1996.
- [3] M. M. Monahemi and M. Krstić, "Control of wing rock motion using adaptive feedback linearization," *Journal of Guidance, Control, and Dynamics*, vol 19, pp. 905-912, 1996.
- [4] Z.-H. Li and M. Krstić, "Maximizing regions of attraction via backstepping and CLFs with singularities," *Systems and Control Letters*, vol. 30, pp. 195-207, 1997.
- [5] F. Ikhouane and M. Krstić, "Robustness of the tuning functions adaptive backstepping design for linear systems," *IEEE Transactions on Automatic Control*, vol. 43, pp. 431-437, 1998.
- [6] R. A. Freeman, M. Krstić, and P. V. Kokotović, "Robustness of adaptive nonlinear control to bounded uncertainties," *Automatica*, to appear, October 1998.
- [7] F. Ikhouane and M. Krstić, "Adaptive backstepping with parameter projection: robustness and asymptotic performance," *Automatica*, vol. 34, pp. 429-435, 1998.
- [8] Z.-H. Li and M. Krstić, "Geometric/asymptotic properties of adaptive nonlinear systems with partial excitation," *IEEE Transactions on Automatic Control*, vol. 43, pp. 419-425, 1998.
- [9] Z. H. Li and M. Krstić, "Optimal design of adaptive tracking controllers for nonlinear systems," *Automatica*, vol. 33, pp. 1459-1473, 1997.
- [10] M. Krstić and Z. H. Li, "Inverse optimal design of input-to-state stabilizing nonlinear controllers," *IEEE Transactions on Automatic Control*, vol. 43, pp. 336-351, 1998.

(accepted)

- [11] M. Krstić, D. Fontaine, P. V. Kokotović, and J. Paduano, "Useful nonlinearities and global bifurcation control of jet engine surge and stall," *IEEE Transactions on Automatic Control*, to appear, 1998.
- [12] M. Krstić and H. H. Wang, "Extremum seeking feedback: stability proof and application to an aeroengine compressor model," accepted for *Automatica*, 1998.
- [13] M. Krstić and P. Tsiotras, "Inverse optimality results for the attitude motion of a rigid spacecraft," submitted to *IEEE Transactions on Automatic Control*, to appear, 1998.

(submitted)

- [14] H.-H. Wang, M. Krstić, and M. Larsen, "Control of deep hysteresis aeroengine compressors—Part I: a Moore-Greitzer type model," submitted to *Journal of Dynamic Systems, Measurement, and Control*, 1996.
- [15] M. Krstić and H.-H. Wang, "Control of deep hysteresis aeroengine compressors—Part II: design of control laws," submitted to *Journal of Dynamic Systems, Measurement, and Control*, 1996.

Refereed Conference Papers

- [16] M. M. Monahemi, J. B. Barlow, and M. Krstić, "Aircraft wing rock control using adaptive feedback linearization: some extensions," *1996 AIAA Guidance, Navigation, and Control Conference*, San Diego, CA, July 1996, AIAA paper 96-3770.
- [17] M. Krstić, "Lyapunov feedback design for stabilization of stall and surge in axial compressors," *1996 ASME International Mechanical Engineering Congress and Exposition*.
- [18] Z.-H. Li and M. Krstić, "Geometric/asymptotic properties of adaptive nonlinear systems with partial excitation," *35th IEEE Conference on Decision and Control*, 1996.
- [19] H.-H. Wang, M. Krstić, and M. Larsen, "Control of deep hysteresis aeroengine compressors—Part I: a Moore-Greitzer type model," *1997 American Control Conference*.
- [20] M. Krstić and H.-H. Wang, "Control of deep hysteresis aeroengine compressors—Part II: design of control laws," *1997 American Control Conference*.
- [21] Z. H. Li and M. Krstić, "Optimal design of adaptive tracking controllers for nonlinear systems," *1997 American Control Conference*.
- [22] Z. H. Li and M. Krstić, "Maximizing regions of attraction in backstepping designs for systems with control singularities," *1997 American Control Conference*.
- [23] M. Krstić and P. Tsiotras, "Inverse optimality results for the attitude motion of a rigid spacecraft," *1997 American Control Conference*.
- [24] Z. H. Li and M. Krstić, "Input-to-state stabilizability is equivalent to solvability of an Isaacs equation," *1997 Conference on Information Science and Systems*.
- [25] M. Krstić and H. H. Wang, "Design of extremum seeking for general nonlinear systems, stability analysis, and application to an aeroengine compressor model," *1997 IEEE Conference on Decision and Control*.

Book Chapters

- [26] P. V. Kokotović and M. Krstić, "A new class of adaptive nonlinear systems," in *Communications, Computing, Control and Signal Processing: A Tribute to Thomas Kailath*, A. Paulraj, V. Roychowdhury, C. D. Schaper, Boston, MA: Kluwer Academic Publishers, 1997.
- [27] P. V. Kokotović and M. Krstić, "Adaptive nonlinear control: a Lyapunov approach," in *Control and Chaos*, K. Judd, A. Mees, K. L. Tao, and T. L. Vincent (Eds.) Boston, MA: Birkhauser, 1997.

6 Additional Data

6.1 Personnel Supported

- Dr. Miroslav Krstic, Assistant Professor, Principal Investigator
- Hsin-hsiung Wang, Graduate Research Assistant
- Zhonghua Li, Graduate Research Assistant

6.2 Transitions

Our extremum seeking scheme for maximization of pressure rise in aeroengine compressors has been discussed with researchers at the *United Technologies Research Center* during two visits by the PI. UTRC is considering taking an exclusive licence for the invention. Point of contact: Dr. Gonzalo Rey, UTRC, Hartford, CT, 860-610-7510.

6.3 Patent Disclosures

Extremum-Seeking Controllers for Maximizing Pressure Rise in Aeroengine Compressors, 1997, filed with University of California.

6.4 Awards

- National Science Foundation CAREER Award, 1996
- O. Hugo Schuck Best Paper Award for 1995 of the American Automatic Control Council
- Best Student Paper Award at the American Control Conference for [21], student author: Zhonghua Li, co-author: Miroslav Krstic.